Models and Precision:

The Quality of Ptolemy's Observations and Parameters

JOHN PHILLIPS BRITTON

GARLAND PUBLISHING INC.

NEW YORK and LONDON

1992

3

Lunar Observations in the *Almagest*: Errors in the Observations and Derived Data

The *Almagest* reports thirty-seven dated observations of the Moon in detail. Twenty-six of them (19 eclipses and 7 occultations) involve only determining the time at which some phase of an event occurred (together with the magnitude in the case of partial eclipses). All the others, except for Ptolemy's determination [*Alm.* v 13] of the Moon's parallax,[1] entail measuring the distance from the Moon to another body at a specific time. Ptolemy [*Alm.* v 12: Toomer 247] also describes his determination of the Moon's greatest northern latitude without mentioning the date of the observations,[2] and he refers [*Alm.* iii 1: Toomer, 135] to two lunar eclipses

[1] The observation was made on +135 Nov 1. Ptolemy finds the Moon's parallax on the meridian at Alexandria to be 1;7° and the Moon's distance from the center of the Earth to be 39;45 Earth radii. Since the Moon was actually near its mean distance, Ptolemy should have found its parallax to be ≈ 0;45°, so that his observation was in error by roughly 0;20°. It is interesting that from this observation Ptolemy deduces a mean distance of the Moon at syzygy, 59 Earth radii, which agrees very well with the modern value, 60.3 Earth radii.

[2] A date for this observation can be inferred from Ptolemy's statement [*Alm.* v 12] that the Moon was simultaneously near the summer solstice (90°) and also near the northern limit of its orbit. To satisfy these conditions, the Moon's ascending node must have been near Aries 0°.

For Ptolemy's time the condition is satisfied in +126 and in +145, the best date for the observation being +126 Aug 3. On this day the Moon culminated about 2 hours before noon with a longitude of 88°, while the position of its ascending node was Aries 0;1°. The observation could have been made a month or so on either side of this date, but the longitude of the Moon at culmination would have been observed by Hipparchus without reporting any details of the observations except the longitudes of Spica which Hipparchus derived from them.[3]

I have limited the following discussion to the dated observations of lunar eclipses, occultations, and elongations, since these observations are completely described, and since Ptolemy's determination of the Moon's parallax is a unique observation which does not fit into any of the other groups. I consider first the eclipses and occultations, which several astronomers have already compared with modern theory and which enable us to evaluate the accuracy of two groups of ancient time-determinations. Then, I discuss the errors in the observations of elongations which involve measurements of both time and arc. These errors have not been investigated previously.

In comparing the lunar observations with modern computations, my objective is to determine, first, the accuracy of the observations which Ptolemy had available to him and, then, the errors in the data Ptolemy uses to determine or demonstrate specific features of his models. The two problems are not quite identical, partly because Ptolemy introduces additional errors in reducing the observations, and partly because the reports themselves are often sufficiently ambiguous to allow several interpretations; indeed, a few of them contain inconsistencies which raise considerable doubt as to their proper interpretation. In general, these difficulties arise only in connection with the observations made by Ptolemy's predecessors and, in particular, the eclipses and occultations where (a) either the time of a specific phase may be uncertain, or (b) the phase associated with a stated time may be uncertain.

The first type of uncertainty can arise either from the vagueness of the time-reference (e.g., 'after rising' in the case of the eclipse of −719 Sep 1 [*Alm.* iv 6: Toomer, 192]) or from an over-determined and inconsistent time-designation (e.g., the occultation of Spica observed by Timocharis in −282 Nov 9 [*Alm.* vii 3: Toomer, 336], where the designation '9½ hours [after Sunset]' and 'just as the Moon was rising' differ by more than an hour). There is also some ambiguity in the meaning of the phrases ὥρας ἀρχομένης and ὥρας ληγούσης, which are frequently applied to times designated in seasonal hours. The question is whether such times should be understood as 'near the beginning' or 'towards the end' of the given hour, or 'at' the beginning or end of the hour. Fotheringham [1915a, 280] has discussed the alternatives and interprets the adjectives, ἀρχομένης and ληγούσης, as referring to the first and last thirds of the stated hour. He then

less satisfactory. This is the earliest date of an observation made by Ptolemy [cf. *Alm.* iv 9: Toomer, 206n54].

[3] See Rome 1937–1938, 6; 1931–1943, iii 828, for discussion of these eclipses.

uses the midpoints of these intervals as the observed time-data. Ptolemy, on the other hand, understands such designations to mean the beginning or end of the hour, and this interpretation was followed by Newcomb [1878, 35].

Another source of uncertainty in the reported times is the occasional ambiguity of the units in which time-intervals are given and of the times of day to which these intervals are referred. Three systems for measuring time are used in describing the observations in the *Almagest*. The most convenient of these, and the one Ptolemy always uses to describe the times of his own observations, states the number of equinoctial hours (1^h = $^1/_{24}$ day) between noon or midnight and the time of the observation. Times given in this system thus require no seasonal correction except for the equation of time.

A variation of this system (historically its predecessor) is encountered in Babylonian astronomical Diaries.[4] Here the time of an event (e.g., Moonrise or the beginning of an eclipse) is given in terms of an interval measured in equinoctial units with respect to Sunset or Sunrise. The Babylonian units of time were the UŠ, equal to $^1/_{6,0}$ days, or 4 minutes, and the DANNA (KAS.BU) or *bēru*, equal to 30 UŠ or 2 equinoctial hours.[5] Times given in this system thus require a seasonal correction, equal to the variation in the time from Sunrise or Sunset to noon (or midnight), as well as the equation of time, in order to reduce them to a uniform system. One observation in the *Almagest* [iv 9: Toomer, 208] which explicitly gives the time in this system is the Babylonian eclipse of −501 Nov 19/20, which 'took place when 6$^1/_3$ equinoctial hours of the night had passed'. Here the unit of measurement is Ptolemy's hour, $^1/_{24}$ of a day, so that the original Babylonian report has been partially modified at least.

The third system of time-measurement, local civil time, was used for reporting nearly all the pre-Ptolemaic observations. In this system the unit of time is the seasonal hour (s.h.), defined as $^1/_{12}$ of the interval from Sunrise to Sunset (or Sunset to Sunrise) of the day on which an event occurs.[6] In this system, the time of an event is the interval between the

[4] Cf. Sachs 1948, 285, for a definition and description of the Babylonian astronomical Diaries. Kugler [cf., e.g., 1907–1924, i 76–77] calls these *Beobachtungstafeln*. Diaries from −651 to +165 have been published in Sachs and Hunger 1988–1989.

[5] For discussions of these units, cf. Kugler 1907–1924, i 25, 272; ii 58–60, 68–71: O. Neugebauer 1955, i 39.

[6] Fotheringham [1932a, 338] and van der Waerden [1951, 20] have discussed evidence of Babylonian computations which seem to use a 'quarter-watch', equivalent to $1^{s.h.}$, as a unit of time. Both conclude that seasonal hours were used in Babylonian astronomy. This is a hazardous inference from very uncertain evidence.

event and Sunset or Sunrise (expressed in seasonal hours). Times in this system, therefore, require a twofold correction in addition to the equation of time in order to reduce them to uniform time, one in the variation in the length of the seasonal hour and the other for the time of Sunset or Sunrise. Ptolemy understands the times of all observations made by others than himself as given in this system, except for the eclipse of +125: Apr 5[7] and the Babylonian eclipses of −522 Jul 16 and −501 Nov 19/20. He generally describes such times in ordinal hours and fractions thereof, and he always explicitly designates these units as seasonal hours in discussing the reductions of these observations.

It is curious that most of the times of the Babylonian eclipses were reduced to this cumbersome system. The reduction serves no astronomical purpose, and it would have been far easier to convert the units given in the Babylonian reports or records directly into equinoctial hours. Since not all of the eclipse-times were reduced to this system, we may ask whether Ptolemy was mistaken in assuming that the times of some of the observations were given in seasonal hours. For this reason I have occasionally included calculations of the errors of the observed times, based on the assumption that the times given refer to equinoctial hours after Sunset, although Ptolemy understands seasonal hours.

Still another uncertainty, which affects several eclipse-observations, arises from Ptolemy's occasional assumption that the report he is citing refers to the midpoint of an eclipse when it says that an eclipse 'took place' at the given time. Instances of this include four of the Babylonian eclipses (−719 Mar 8, −522 Jul 16, −501 Nov 19/20, and −490 Apr 25) and the eclipse of +125 Apr 5. Nevill [1906, 2] and, following him, Cowell [906, 523] and Fotheringham [1920a, 578–579] have interpreted all times but that of −719 Mar 8 as specifying the beginning of the eclipse. Since there is at least one case, −381 Dec 12, in which Ptolemy takes the same vague description to refer to the beginning of the eclipse, and since the Babylonian diaries generally state the time of the beginning, but not the midpoint, of eclipses, I shall consider the possibility that Ptolemy may have been mistaken about the phase in these instances.

---

[7] The earliest observation in the *Almagest* which Ptolemy explicitly claims to have made himself is his observation of an opposition of Saturn on +127 Mar 26. Since his observation of the Moon's extreme latitude probably antedates this [see 49n3 above], and since the time reported for this eclipse is given in equinoctial hours relative to midnight, it is quite possible that he himself observed the eclipse of +125 Apr 5 [*Alm.* iv 9: Toomer, 206]. He says only, however, that the eclipse was observed in Alexandria.

Finally, I should note that throughout this discussion the term 'error' (alternatively, Δ) denotes simply the difference between a datum calculated from modern theory and that reported by Ptolemy, always in the sense of a correction to Ptolemy's datum. Thus, the term embraces all sources of error in a given datum such as errors of measurement, recording, reduction, transmission, interpretation, and so on, in addition to any error in the modern theory on which the calculation is based.

ECLIPSE-OBSERVATIONS

In this section I consider Ptolemy's reported eclipse-observations individually in chronological order. Table 3.1 contains the following computed data used to determine the errors of these observations:

col. 1 Julian civil date when the midpoint of the eclipse occurred.

col. 2 Local apparent time (midnight epoch) of eclipse-midpoint for:
Babylon ($2;58^h$) E: 32;30° N
Alexandria ($2;0^h$) E: 31;12° N
Rhodes ($1;53^h$) E: 36;24° N.
For eclipses nos. 1 through 15, the times are taken from P. V. Neugebauer [1934, 13]; the times of eclipses nos. 16 through 19 are from Newcomb [1878, 42].

col. 3 Half-duration of the eclipse, for eclipses 1 through 15 from P. V. Neugebauer [1934]; for eclipses 16 through 19 from Cowell [1906, 526]. Cowell's computed durations are nearly identical with Newcomb's when the latter are corrected for the error in Hansen's argument of latitude.

col. 4 Correction to the mean tabular elongation at the time given in col. 2 which is needed to reduce the tabular elongation to my elements. For eclipses nos. 1–15 and 16–19, these are from appendix 2, Table A2.1 and Table A2.3, respectively.

col. 5 Velocity of the Moon's elongation at the time of the eclipse in seconds of arc *per* minute of time, as computed from Newcomb [1878, 41].

col. 6 Correction to the tabular time in col. 2 obtained by dividing col. 4 by col. 5. (The sign of the correction is positive, since the negative correction to the elongation at tabular eclipse-midpoint means the Moon must still travel ΔD to reach eclipse-midpoint.)

col. 7 Corrected local apparent time (midnight epoch) of eclipse-midpoint (col. 2 + col. 6).

No.	1 Date and Place	2 $T'(M)$	3 Dur.	4 ΔD	5 $\frac{\Delta D}{\Delta t}$	6 ΔT	7 $T(M)$	8 $T(B)$	9 $T(E)$	10 $\approx L_s$	11 $^1/_2 N$	12 Mag.
1	−720 Mar 19 Babylon	21.5^h	1.9^h	−555″	29.1″/m	$+0;19^h$	$21;49^h$	$19;55^h$		351.5°	$6;\ 9^h$	10.2^d
2	−719 Mar 8 Babylon	23.6	0.7	−555	27.1	+0;20	23;56	23;14		340.7	6;20	1.5
3	−719 Sep 1 Babylon	20.0	1.2	−555	35.4	+0;15	20;15	19; 3		159.9	5;30	6.1
4	−620 Sep 22 Babylon	5.25	0.85	−510	27.2	+0;19	5;34	4;43		24.4	5;35	2.1
5	−522 Jul 16 Babylon	23.65	1.35	−465	27.6	+0;17	23;56	22;35		106.6	4;58	6.1
6	−501 Nov 20 Babylon	0.1	0.8	−460	27.0	+0;17	0;23	23;35		231.9	6;51	2.1
7	−490 Apr 25 Babylon	22.75	0.65	−455	31.4	+0;14	22;59	22;20		28.5	5;32	1.7
8	−382 Dec 23 Babylon	0.0	0.9	−410	34.8	+0;12	8;12	7;18		267.0	7; 6	3.0
9	−381 Jun 18 Babylon	21.15	1.35	−410	27.5	+0;15	21;24	20; 3	$22;45^h$	80.5	4;57	5.9
10	−381 Dec 12 Babylon	23.05	1.75	−410	35.5	+0;12	23;15	21;30		256.2	7; 5	18.2
11	−200 Sep 22 Alexandria	18.9	1.5	−340	28.7	+0;12	19;12	17;42	20;42	176.0	5;56	8.5
12	−199 Mar 20 Alexandria	0.8	1.8	−340	32.0	+0;11	0;59	23;11		355.4	6; 5	16.0
13	−199 Sep 12 Alexandria	2.35	1.85	−340	32.3	+0;11	2;32	0;41		165.0	5;45	19.3
14	−173 May 1 Alexandria	1.8	1.3	−330	35.4	+0; 9	1;57	0;39	3;15	35.7	5;26	7.4
15	−140 Jan 27 Rhodes	21.65	0.85	−320	35.6	+0; 9	21;48	20;57		304.5	6;49	2.8
16	+125 Apr 5 Alexandria	20.75	0.77	−260	32.2	+0; 8	20;53	20; 7		14.3	5;52	1.8
17	+133 May 6 Alexandria	22.93	1.77	−257	28.2	+0; 9	23; 5	21;19		44.2		12.9
18	+134 Oct 20 Alexandria	22.93	1.57	−256	29.0	+0; 9	23; 5	21;31		206.3		10.1
19	+136 Mar 6 Alexandria	3.37	1.35	−254	34.6	+0; 7	3;29	2; 8		344.6		5.5

Table 3.1. Data for Comparing Eclipses

col. 8 Corrected local apparent time of the beginning of the eclipse, i.e., col. 7 − col. 3.

col. 9 Corrected local apparent time of the end of the eclipse, for those for which Ptolemy gives the time of the end, i.e., col. 7+ col. 3.

col. 10 Approximate longitude of the Sun from Newcomb [1878, 41].

col. 11 Half the length of the night in equinoctial hours, computed accurately from Ptolemy, *Alm.* ii 8 for the latitudes assumed above in col. 2. For the apparent half-length of the night, subtract 2^m from these values to correct for refraction.[8]

col. 12 Magnitude of the eclipses: nos. 1 through 15 are from P. V. Neugebauer [1934]; nos. 16, 18, 19 are from Fotheringham [1909c, 668]; and no. 17 is from Oppolzer [1962].

The precision of the times from P. V. Neugebauer is $\pm 0;3^h$, excluding the uncertainty of the secular accelerations. If we assume an uncertainty of $\pm 0.3'' T^2$ in the secular acceleration of the mean elongation used in the comparison, the corresponding errors in the eclipse-times would be $\approx \mp 0;6^h$ at −600 and $\mp 0;4^h$ at −100. Further, the uncertainty in the argument of latitude can increase the error in the time of an initial or terminal phase by $\pm 0;2^h$. Finally, the error in the computed time of Sunrise or Sunset is estimated to be $\pm 0;2^h$. The probable error of a computed time, therefore, will be roughly $\pm 0;6.5^h$.

Eclipse 1. −720 Mar 19 — *Alm.* iv 6: Toomer, 191

1 Mardokempados: 29/30 Thoth[9]

> The eclipse began, [the report] says, well over an hour after Moonrise, and was total.

[8] Fotheringham [1915a, 381] and Schoch [1926, 32] understand 'Sunrise' and 'Sunset' to mean the appearance or disappearance of the Sun's upper rim, in which case the half-length of the night should be further reduced by $0;1^h$. I cannot determine on what basis they make this assumption, and I have assumed rising or setting to refer to the center of the body in question.

[9] To be read as 'year 1 of the reign of Mardokempados, on the night between Thoth 29 and Thoth 30'. Ptolemy uses a continuous Egyptian calendar, the epoch of which is Thoth 1, Nabonassar 1 (= −746 Feb 26, JDN 1488638). The number of days between an event and this epoch can be obtained by first finding the number of intervening Egyptian years (each of 365 days) from Ptolemy's List of Reigns [Ginzel 1906–1914, i 139] and then adding the number of days between

Lunar Eclipse-Data	Computed[a]	Ptolemy	Δ
Sunset (Babylon)	17;53^h	18; 0^h	−0; 7^h
Moonrise (Babylon)[b]	17;40		
Beginning (Babylon)	19;55	19;30	+0;25
Midpoint at Babylon	21;49	21;30	+0;19
Alexandria	20;51	20;40	+0;11
Magnitude	18.2^d	(21.6)^d	(−3.4)^d

[a] All computed times are given in the local apparent time (midnight epoch) of the place indicated. The times of risings and settings refer to the center of the body indicated and are corrected for refraction.

[b] Newcomb, following Zech [1851, 13], puts Moonrise at 17;53^h mean time. This is consistent with 17;40^h local apparent time when refraction is taken into account.

Eclipse No. 1: −720 Mar 19

Ptolemy assumes the night at Babylon was 12^h long and that the eclipse began 1½^h after Sunset 1;45^h after Moonrise. He also assumes the eclipse was central and computes the duration as 4^h.[10]

The time given for this eclipse has caused substantial difficulties for modern investigators. Nevertheless, there is general agreement that Ptolemy's

---

Thoth 1 and the date of the event. The order of the months is:

I	Thoth	VII	Phamenoth
II	Phaophi	VIII	Pharmuthi
III	Athyr	IX	Pachon
IV	Choiak	X	Payni
V	Tybi	XI	Epiphi
VI	Mechir	XII	Mesore.

Each has 30 days and Mesore is followed by 5 epagomenal days. In computing the number of days since Thoth 1, it must be remembered that Thoth 1 counts as day zero. Furthermore although Ptolemy's calendar assumes a midnight epoch, he uses noon on Thoth 1, Nabonassar 1 as the epoch of his mean motion tables. This has the advantage that all observations made at night can be reduced on a uniform basis without considering whether they were made before or after midnight.

[10] Indeed, according to Manitius [1912–1913, i 433n28], the duration computed from Ptolemy's tables, assuming a central eclipse, is 3;59,45^h. See also Toomer, 191n30.

estimate of the elapsed time since Moonrise, 1½ hours, is the maximum time consistent with the description well over one hour, although Ptolemy applies this interval to Sunset rather than Moonrise. Newcomb [1878, 35–36] assumes that the report indicates an interval of between 1¼ and 1½ hours after Moonrise, and finds the difference between the observed and computed time to be:

Lunar Datum	Computed	Observed	Δ
Time since Moonrise	$2;15^h$	$1;22^h$	$+0;53^h$

On the other hand, if we follow Ptolemy's interpretation applied to accurate Sunrise, the error (Δ) is $+0;32^h$.

Kugler [1907–1924, ii 68] has suggested that the time accepted by Ptolemy can be explained by assuming that the original report said only that the eclipse occurred in the first watch and that its total phase ended before the end of the first watch; this would require the eclipse to have begun less than $1;40^h$ after Sunset. Although Kugler correctly remarks that the unit, $1^h = 1/24$ day, is not a Babylonian unit [see 50n6, above], his explanation still does not account for the description Ptolemy quotes. An alternative explanation is that whoever transmitted this report mistakenly translated 1 KAS.BU (DANNA = double hour) into 'well over one hour'. If so, the time, 1 double hour after Moonrise, would agree very closely with the computed time. If Ptolemy's report more or less accurately represents the Babylon account, however, the error is nearly an hour.

Eclipse 2. −719 Mar 8 *Alm.* iv 6: Toomer 192

2 Mardokempados: 18/19 Thoth

> The [maximum] obscuration, [the report] says, was 3 digits from the south exactly at midnight.

Kugler [1907–1924, ii 69] assumes that the original report may have indicated that the greatest phase occurred when the Moon was on the meridian, and notes examples from other texts to demonstrate this possibility. Since the eclipse was of short duration ($1;24^h$), determining the time by reference to the meridian would make the observation's probable error much less than we might otherwise expect from the somewhat vague description.[11] In any

[11] Newcomb [1878, 36] assumes a probable error of $\pm 40^m$, equal to half the duration of the eclipse. As a result, he gave the eclipse very little weight in his subsequent analysis.

case, the stated time agrees very well with the computed time if we assume that the observed time referred to mid-eclipse.

Lunar Eclipse-Data	Computed	Ptolemy	Δ
Midpoint at Babylon	23;56^h	24; 0^h	−0; 4^h
Alexandria	22;58	23;10	−0;12
Magnitude	1.5^d	3.0^d	−1.5^d

Eclipse No. 2: −719 Mar

Eclipse 3. −719 Sep 1 *Alm.* iv 6: Toomer, 192

2 Mardokempados: 15/16 Phamenoth

> The eclipse began, [the report] says, after Moonrise, and the [maximum] obscuration was more than half from the north.

Ptolemy concludes that the eclipse began at least half but less than one (equinoctial) hour after Moonrise, implying that a smaller or greater interval would have been specifically mentioned. He then adopts half an (equinoctial) hour after Sunset (≈ 0;40^h after Moonrise) as his beginning time. Newcomb [1878, 36] assumes 0;25^h after Moonrise as most probable. Ptolemy computes the duration to be 3 hours, equivalent to an assumed magnitude of 8^d by his tables. Ptolemy's assumption that the eclipse began half an hour after Sunset is in excellent agreement with the computed time.

Lunar Eclipse-Data	Computed	Ptolemy	Δ
Moonrise (Babylon)	18;24^h		
Sunset (Babylon)	18;23	18;30	
Beginning (Babylon)	19; 3	19; 0	+0; 3^h
Time since Moonrise	0;39	0;30[a]	+0; 9[a]
Sunset	0;31	0;30	+0; 1
Midpoint at Babylon	20;15	20;30	−0;15
Alexandria	19;17	19;40	−0;25
Magnitude	6.1^d	[8.0^d]	[−1.9^d]

[a] With Newcomb's estimate of 0;25^h, the error is +0;14^h.

Eclipse No. 3: −719 Sep 1